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Magnetic Microfields of Random Currents

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The distribution of magnetic microfields in systems with stochastic currents including plasmas, solids, and electrolytes is studied. It is shown that the random currents generate magnetic microfields distributed similar to the Holtsmark distribution for the electrical fields. Several relevant physical effects provided by this magnetic microfield and applications including plasma displays, microelectronic chips and currents in biological objects are discussed.

1 Introduction - basic concepts and Holtsmark distribution

The first investigation of the electrical microfields which are created by the charges in a plasma was performed in 1919 by J. Holtsmark [1]. He succeeded to derive the distribution of the electric microfield at a fixed neutral point in ideal plasmas [1]. The calculation of the magnetic field distribution differs from that for the electric field since the elementary magnetic field generated by charged particles depends on their velocity and is, in general, much weaker. The electric microfields influence many microscopic processes, some of them are listed below, a summary has been given in [2]. So far the magnetic microfields in plasmas did not attract such large interest as the electrical microfields, a pioneering study is due to Kalman [3]. This is due to several factors: First of all, the expected average value of the magnetic microfields is in "normal" plasmas rather small in comparison with the electric field. For thermal systems we may expect a factor v_T/c, where v_T is the average thermal velocity of the plasma particles. It follows that the ionic contribution to the total microfield in a thermal plasma is by the factor $(m/M_i)^{1/2}$ smaller than the electronic component. Here M_i is the ion mass, for simplification we consider here only one species of ions. At second, the expected time interval, in which the magnetic microfield can be expected to be stable, is about the inverse electron plasma frequency ${\omega_{pe}}^{-1}$. This rather small time of field stability decreases the possible action of the magnetic field on microscopic processes. Finally, the magnetic field-ion interaction is by a factor $\alpha^{1/2}$ lower, in comparison with the electric interaction of fields with the ions. Here α is the fine structure constant (compare also the relation of Zeeman shift to Stark shift). On the other hand many properties of the magnetic microfields, like the dependence from the point of measurement (neutral or charged) as well as the dependence from the plasma non-ideality, are similar to those observed for electric microfields.

In general however, the magnetic microfield in plasmas is small and has some specific properties in comparison to the electric microfield [3–5]. For most applications the magnetic interactions of currents are not relevant, opposite cases as the problem of magnetic microfield interactions with random currents in Z-pinches and other electroneutral non-equilibrium plasmas are not studied here.

However, there are specific cases, where the magnetic microfields are relevant. In a recent study several effects of magnetic microfields acting on physical properties have been found for plasmas, solids and biological objects [6].

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It was shown, that there are several physical and technical situations, where discrete current elements whith random distribution create strong magnetic microfields. Most often these random currents are not of thermal origin but of mesoscopic character. Typical examples are the currents in a plasma display, in microelectronic chips and in biological membranes. The present study is concentrated on such situations. Our most important example for magnetic microfields in plasmas concerns the plasma displays which have nowadays a large technological relevance. In this case the currents are directed perpendicular to the plane of the display. The currents are not random in the mathematical sense, however they are practically of such a high complexity that we may model them as random. Our first application will refer to this case.

The magnetic microfields in solids differ from the microfield in plasmas mainly because of the different character of the velocity distributions of the electrons. In plasmas we have Boltzmann distributions and in solids we have Fermi distributions corresponding to degenerated electrons. Our methods permit to consider both types of distributions. However in this work we are mainly interested in mesoscopic random currents of non-thermal origin. As a possible application we will discuss the currents and fields in microchips, again using the assumption that the current distribution in large chips is rather complex and almost stochastic.

A different field of interest are electrolytes, which also are quite relevant for technical applications and for the functioning of biological objects (living systems). Due to the thermal motion of the ions in electrolytes we find also magnetic microfields, which sometimes are sufficiently strong to be observed. For comparison, the characteristic Earth magnetic field is about 1 CGS, and the sensitivity of living object s to such fields is known. Again we concentrate here in our context, on current distributions of non-thermal origin. As examples we consider the complex currents in membranes, tissues, the brain etc. The microscopic currents in such biological systems, like the membranes, tissues and the brain are in most cases highly complex and may be, in a first approximation, considered as stochastic ones. The microscopic currents and fields inside such constituents of living objects as mithochondriae, organells, erytrocytes in blood, etc., could be even larger than the currents and fields in macroscopic membranes and tissues.

As a last example for an application we will consider the conducting membranes in hydrogen fuel cells.

2 Microscopic, discrete current and field approaches

There are two essentially different methods to calculate the magnetic field distribution, the microscopic approach and the macroscopic field approach. For convenience we use for both approaches different units: CGS (for the microscopic approach), and SI (for macroscopic and mesoscopic field studies).

(i) The microscopic approach:

We assume that the system consists of N moving charges, then we calculate the contribution of every individual charge and sum up over all particles. If the i-th particle with the charge q_i is located at the point $\mathbf{r}_i$ and has the velocity $\mathbf{v}_i$, the nonrelativistic value of the magnetic field at a neutral-point located at the origin of coordinates is:

$$\mathbf{H}_i = q_i[\mathbf{v}_i\mathbf{r}_i]/c{r_i}^3. \tag{1}$$

Here r_i is the absolute value of the vector $\mathbf{r}_i$ and c is the speed of light. The total magnetic field is given as the sum of all individual contributions of the N charged particles (1)

$$\mathbf{H} = \sum_i^N \mathbf{H}_i = \sum_i q_i \frac{[\mathbf{v}_i\mathbf{r}_i]}{c{r_i}^3}. \tag{2}$$

The motion of point charges like electrons can be defined in such media like plasmas and solids (conductors and semiconductors).

(ii) Mesoscopic and macroscopic field approach:

We assume now that the matter which we consider is characterized by a distribution of discrete stochastic currents l where l is an element of current. Examples have been given above. The Biot-Savart-Laplace law for a magnetic induction created by an element of current reads

$$\mathbf{B} = \mu\mu_0 \frac{[\mathbf{l}_i\mathbf{r}_i]}{{r_i}^3} \tag{3}$$

where the i-th element of current is located at the point $\mathbf{r}_i$, μ is a magnetic permeability, μ_0 is the magnetic permeability of vacuum. The current element is the product of the scalar value of the current propagating through this element and the space vector determining the direction of the current propagation. In our model, both values the elementary current and the vector (direction, length) are assumed to be random. The total magnetic induction is then the sum of (3) over all current elements

$$\mathbf{B} = \sum_i^N \mathbf{B}_i = \sum_i^N \mu\mu_0 \frac{[\mathbf{l}_i \mathbf{r}_i]}{{r_i}^3}. \tag{4}$$

Of course, the expression (4) has a physical meaning only if current elements with discrete character are clearly defined.

If the currents are continuously distributed and characterized by a current distribution $\mathbf{j}(\mathbf{r})$, the vector potential is obtained as solution of Maxwells equation

$$\mathbf{A}(r) = \mu\mu_0 \int d\mathbf{r}' \frac{\mathbf{j}(r')}{|r - r'|}. \tag{5}$$

Alternatively we may write

$$\mathbf{B}(r) = \mu\mu_0 \int d\mathbf{r}' \mathbf{j}(r') \times \frac{\mathbf{r} - \mathbf{r}'}{|r - r'|^3}. \tag{6}$$

Having in mind the examples discussed above we concentrate here on the case where the currents may be modelled by a distribution of discrete (line-shaped) currents and the field may be calculated from eqs. (3-4).

3 Distribution of magnetic microfields

3.1 Isotropic systems

We formulate the problem in terms of current elements assuming that the i-th current elements are

$$\mathbf{l}_i = i_i \mathbf{L}_i,$$

where i_i is the elementary current through some elementary "tube" or "wire" with the length L and definite direction along this "tube" or "wire". In this case, the magnetic induction (in SI-system of physical units) is determined by (3), and the total magnetic induction in the center of coordinates will be determined by (4). The distribution function $W(\mathbf{B})$ is expressed through a probability to find the first current element at the volume $d\mathbf{r}_1$ around the point $\mathbf{r}_1$ with the amplitude $\mathbf{l}_1$, the second current element at the volume $d\mathbf{r}_2$ around the point $\mathbf{r}_2$ with the amplitude $\mathbf{l}_2$, etc up to the N-th current element $P_N(\mathbf{r}_1, ..\mathbf{r}_N, \mathbf{l}_1, ...\mathbf{l}_N) d\mathbf{r}_1...d\mathbf{r}_N d\mathbf{l}_1...d\mathbf{l}_N$ as

$$W(\mathbf{H}) = \int \delta(\mathbf{B} - \sum \mu\mu_0 [\mathbf{l}_i \mathbf{r}_i]/{r_i}^3) P_N(\mathbf{r}_1, ..\mathbf{r}_N, \mathbf{l}_1, ...\mathbf{l}_N) d\mathbf{r}_1...d\mathbf{r}_N d\mathbf{l}_1...d\mathbf{l}_N. \tag{7}$$

The summation is performed over all N current elements in a considered system (plasmas, solids, biological objects) that created the field at the point of interest. In the case of isotropic 3D- distributions we find

$$W(B) = 4\pi B^2 W(\mathbf{B})$$

and in the case with cylindrical symmetry

$$W(B) = 2\pi B_\rho B_z W(\mathbf{B}).$$

Here B_ρ is the transversal magnetic induction, B_z is the longitudinal one. Let us consider the simplest case of a distribution function for the microfield produced by statistically independent current elements. In this case, the function P_N can be expressed as a product of N independent functions

$$P_N(\mathbf{r}_1, ..\mathbf{r}_N, \mathbf{l}_1, ...\mathbf{l}_N) = \Pi_i^N P_1(\mathbf{r}_i, \mathbf{l}_i) \tag{8}$$

The Fourier component of the δ-function is:

$$\delta(\mathbf{B}-\mathbf{B}') = (2\pi)^{-3}\int d\mathbf{K}\exp[i\mathbf{K}(\mathbf{B}-\mathbf{B}')].$$

In this way we get for the distribution of the microfield generated by random current elements

$$W(\mathbf{B}) = \frac{1}{(2\pi)^3}\int d\mathbf{K}\exp(i\mathbf{KB})\int d\mathbf{r}_1...d\mathbf{r}_N d\mathbf{l}_1...d\mathbf{l}_N \Pi_i^N P_1(\mathbf{r}_i,\mathbf{l}_i)\exp(-i\mu\mu_0\frac{(\mathbf{K}[\mathbf{l}_i\mathbf{r}_i])}{{r_i}^3}). \quad (9)$$

Here, the multiple integral for statistically independent current elements is just the product of N six-dimensional integrals:

$$I_i(\mathbf{K}) = \int d\mathbf{r}_i d\mathbf{v}_i P_1(\mathbf{r}_i,\mathbf{v}_i)\exp(-i\frac{(\mathbf{K}\mu\mu_0[\mathbf{l}_i\mathbf{r}_i])}{{r_i}^3}). \quad (10)$$

Let us consider now the isotropic case. We have to evaluate the last six-dimensional integral over $\mathbf{r_i}$ in polar coordinates. Our first assumption is, that all current elements are independently located in space (in fact, this is an analogy of the ideal plasma approximation),

$$P_1(\mathbf{r}_i,\mathbf{l}_i) = V^{-1}P_1(\mathbf{l}_i).$$

Then, evaluating the integral $I_i(\mathbf{K})$ over the directions (the angles) of the radius vector, we find

$$I_i(\mathbf{K}) = \int_{\mathbf{l}_i} d\mathbf{l}_i P_1(\mathbf{l}_i)\int_0^R dr_i r_i^2 \sin(\frac{K\mu\mu_0 l_i \sin(\alpha_i)}{{r_i}^2})/(\frac{K\mu\mu_0 l_i \sin(\alpha_i)}{{r_i}^2}). \quad (11)$$

Here R is the radius of the plasma. We assume $V \to \infty$ (or $R \to \infty$), the total number of current elements $N \to \infty$, the density $N/V = n \to const$. Making several partial integrations over r_i in (11), we find

$$I_i(\mathbf{K}) = 1 - \frac{4\sqrt{2}\pi^2 K^{3/2}{\mu\mu_0}^{3/2}\Gamma(7/4)}{15V} < l_i^{3/2} > \quad (12)$$

where $< l_i^{3/2} >= \int_0^\infty dl_i l_i^{3/2} P_1(l_i)$, $\Gamma(x)$ is Euler's gamma-function. Supposing that all $< l_i^{3/2} >$ have the same value and making the standard transition to an exponential limit

$$[1 - 4\sqrt{2}\pi^2 K^{3/2}{\mu\mu_0}^{3/2}/15V < l^{3/2} >]^N \to \exp(-(KB_0)^{3/2}),$$

we get the characteristic value of the random magnetic induction

$$B_0 = \frac{2^{5/3}\pi^{4/3}}{15^{2/3}}(\Gamma(\frac{7}{4}))^{2/3}\frac{\mu\mu_0 n < l^{3/2} >^{2/3}}{c} \quad (13)$$

(n is the density of current elements). This way, we get finally the magnetic microfield distribution function expressed in terms of the normalized field value $\beta_1 = B/B_0$

$$W(\beta_1) = \frac{2\beta_1}{\pi}\int_0^\infty x\sin(\beta_1 x)\exp(-x^{3/2})dx. \quad (14)$$

This is a Holtsmark distribution simular to the electric case [1, 3, 5]. As pointed out above, in thermal plasmas the expected average value of this magnetic microfield is rather small in comparison with the electric field. In general we may expect for thermal systems a factor v_T/c, where v_T is the average thermal velocity of plasma particles. Consequently, the ionic input into the total microfield is smaller than the electronic component by the factor $(m/M_i)^{1/2}$; this consideration refers to equilibrium plasmas. Here M_i is the ion mass (for simplification we consider only one sort of ions). The fields may be much higher in turbulent plasmas [2].

Similar arguments apply to electrolytic systems. The situation is quite different for systems with random currents of non-thermal origin. The microfields created by thermally moving ions in electrolytes (concentration$\sim 10^{15}$ cm^{-3}, room temperatures) is about $4-5$ orders less than the earth magnetic field [6].

For biological objects (like cell membranes) there might be several sources of mesoscopic currents. Of particular importance are the proton pumps and pumps for different ions like Na, K, and Ca. These pumps are generating little currents everywhere in the body. The proton pumping is generated by the H-ATP-ase (F1/F10 ATP synthase in chloroplasts). The spatial distribution of these units in the living body is practically uniform, and we can expect that the magnetic microfield distribution will be of Holtsmark-type. The potential over membranes is about 0.1 V, the resistence $100 - 1000$ Ohm $\cdot$ cm^2, the width of H-ATP-ase molecules is about $3 \cdot 10^{-8}$ m. Therefore the total current in the pumping unit is about 10^{-10} A, the density of H-ATP-ase molecules is $\sim 10^{20}$ m^{-3} and the characteristic magnetic induction $B_0 \sim 10^{-8} - 10^{-9}$ T. This corresponds to magnetic field magnitudes of about $3 - 4$ orders less then the magnitude of the earth magnetic field. The corresponding magnetic induction provides a "basic" magnetic chaos inside cells and organells like mithochondria which is in general much smaller than the earth magnetic field. However the contributions of fluctuations due to the "fat tail" of Holtsmark distribution (14) may exceed the earth magnetic field and have values of about $10^{-7} - 10^{-9}$ T . These are just first estimates, biological systems require a more detailed discussion.

3.2 The anisotropic case

We study now several examples where the currents are bound to sheets or displays, i.e. essentally to 2D-objects. Our aim is to calculate magnetic microfields for such a special system like plasma screens, large microchips, and conducting membranes (in hydrogen fuel cells, for example). The plasma display (screen in TV-sets) is a planar grid structure with several millions of knots of elementary discharges. The strength of the elementary discharges (currents) has nearly random character, since the current through a knot is determined by a very complex TV-picture. Therefore we can model it in a first approximation as stochastic. The magnetic induction in each screen point can be determined by eq. (4). Every elementary current flows only along a specific line (channel) perpendicular to the screen (along the axis Z, for example). This way, the isotropic calculations corresponding to eq. (14) cannot be applied. The space distribution of knots on plasma screens is uniform (even regular). It means that a current element can be found at each point of the screen. We will assume that the space distribution of current elements is $P_N = 1/V^N$. Let us rewrite the expression (11) making the standard trick to introduce the unity under the integral:

$$I_i^z(\mathbf{K}) = 1 - \frac{1}{V}\int_{l_i} P_1(l_z i)\int_V d\mathbf{r}_i(1 - \exp(-i\mu\mu_0\frac{(\mathbf{K}[\mathbf{l}_i\mathbf{r}_i])}{r_i{}^3})). \tag{15}$$

We integrate by parts assuming that there is no dependence on the Z coordinate. This means that the screen or other objects which we consider have to be very thin. Then eq. (15) is reduced to the form

$$I_i^z(\mathbf{K}) = 1 - \frac{\mu\mu_0 < l > K_\rho}{2V}. \tag{16}$$

There is no Z-dependence. Further K_ρ is the Fourier-decomposition of the vortex magnetic field in the screen plane, and $< l >$ is the mean value of current elements, which is determined by the function $P_1(l_z i)$. Due to the geometry of the screen all the vectors along current elements have the same length L. Then we find $< l > = < i > L$ where $< i >$ is the mean current. Further $N < i > L/V = j$ is the mean current density, which can be measured. The standard transition to an exponential limit gives

$$(I_i^z(\mathbf{K}))^n = (1 - \frac{\mu\mu_0 < l > K_\rho}{2V})^N \rightarrow \exp(-\frac{\mu\mu_0 K_\rho L j}{2}).$$

We return to the distribution function $W(\mathbf{B})$, denote the characteristic "membrane" microfield by $B_M = \mu\mu_0 L j/2$, and write

$$W_M(\mathbf{B}) = \frac{\delta(B_z)B_M}{2\pi\sqrt{B_\rho^2 + B_M^2}^3}.$$

This way we find for the distribution function of the amplitude of the vortex microfield in the screen plane

$$W_M(B_\rho) = \frac{B_M}{\sqrt{B_\rho^2 + B_M^2}^3}. \tag{17}$$

The total current through plasma screens can achieve several A while L is up to 1 mm. Therefore B_M is about $10^{-6} - 10^{-7}$ T. The corresponding magnetic field is about $1 - 2$ orders less than the earth's magnetic field. Nevertheless, large fluctuations are possible and should be taken into account.
Another example of parallel current propagation through a thin layer is the membrane in a hydrogen fuel cell element (for example in the well-known NAFION membrane). The current through the membrane is provided by protons, the current density can achieve several A/cm^2. This gives a B_M of the order of $10^{-5} - 10^{-6}$ T, i.e. about or slightly less than the earth's magnetic induction. The "fat tail" of the distribution (17) may be responsible for an observable decrease of the membrane proton conductivity. Indeed, the Lorentz force of the magnetic microfield may influence the proton path and lead to deviations from the direct propagation through the membrane (this is an analogue to the Hall effect). The pathway of the protons can be written as

$$L_B \simeq L\sqrt{1 + \omega_H^2 \tau_{corr}^2},$$

where ω_H is the Larmor frequency of the microfield due to protons, τ_{corr} is the microfield correlation time (it may be estimated as the proton passage time through the membrane). Since $\omega_H \sim 10^{-3}$ Hz, the membrane resistance is inversely proportional to L_B, it means that the membrane resistance R may be essentially increased by the magnetic microfield. Note, that the final resistance can be calculated by averaging $R = R_0/\sqrt{1 + \omega_H^2 \tau_{corr}^2}$ with the function (17). The final result provides the nonlinear (over total current) resistance of the fuel cell.

Another interesting anisotropic situation arises when the current propagation is parallel to a plane, i.e. it is bound to a thin sheet or a layer. This situation we meet e.g. in the modern microchips. Indeed, microchips are structured as a set of planar layers. The raw material for chips may have the characteristic size of one layer of about 5 cm, the total width may be about 0.5 cm. The characteristic size of microchip structures is recently about 0.09μm; the transition to 0.065μm structures is expected soon. This means that the total number of elements in one layer is $\sim 3 \cdot 10^{12}$. The current through one structure element changes rapidly in time: a characteristic frequency of recent microchips is about $3 \cdot 10^9$ Hz. Of course, the internal network structure of the microchips determines the concrete dependence of the current in one element. Correlations are due to the externally given structure and due to Kirchhoff's laws. Nevertheless, we may assume in first approximation that the current elements are independent and stochastically distributed. In part this is due to the existence of small-scale current correlations inside microchips. Moreover, the current elements inside microchipes are fixed, they cannot move. At the same time, the current through an element can change in a comparably wide range: from zero to nanoamperes. Due to the complexity of the whole structure, the currents can be modelled in a first approximation by a random distribution (ranging from zero to $i_{max} \sim 10^{-8} - 10^{-9}A$). At the moment when the chip is active, i.e. a current flows through the chip, the conducting elements represent just a few percent of the wires on the microchip. The other little wires do not carry a current. Following these physical arguments an approach by the methods developed above to determine the "magnetic chaos" as a result of stochstic currents inside the microchip may be justified.
The electric contacts between layers are much less dense than the contacts (elements) inside the layer. Thus we may neglect the currents between layers to simplify this way the model. The currents (elements) inside a layer can be directed either along the X-axis or along the Y-axis only. It means that the function P_N to find the first current element $\mathbf{l}_1$ at the volume $d\mathbf{r}_1$ around the point $\mathbf{r}_1$, the second current element $\mathbf{l}_2$ at the volume $d\mathbf{r}_2$ around the point $\mathbf{r}_2$, etc. up to the $N-$th element can now be written as

$$P_N(\mathbf{r}_1, ..\mathbf{r}_N, \mathbf{l}_1, ...\mathbf{l}_N) = \Pi_{i=1}^{N_x} P_1(\mathbf{r}_i, l_{ix}) \Pi_{j=1}^{N_y} P_1(\mathbf{r}_j, l_{jy}) \Pi_{k=1}^{N_z} P_1(\mathbf{r}_k, l_{kz}),$$

when we have totally N_x elements directed along the X-axis, N_y elements directed along the Y-axis, and N_z elements directed along the Z-axis, $N_x + N_y + N_z = N$. Then the general shape of the distrubution fuction of a magnetic microfield is

$$W(\mathbf{B}) = \frac{\mu\mu_0}{(2\pi)^3} \int_{-\infty}^{\infty} d\mathbf{K} \exp[i(\mathbf{KB})] \Pi_{i=1}^{N_x} \int d\mathbf{r}_i dl_{ix} \exp(-i(\mu\mu_0 l_{ix} \mathbf{K}[\mathbf{r}_i \mathbf{n}_x])/r_i^3) P_1(\mathbf{r}_i, l_{ix})$$

$$\cdot \Pi_{j=1}^{N_y} \int d\mathbf{r}_j d_{jy} \exp(-i(\mu\mu_0 l_{jy} \mathbf{k}[\mathbf{r}_j \mathbf{n}_y])/r_j^3) P_1(\mathbf{r}_j, l_{jy})$$

$$\cdot\Pi_{k=1}^{N_z}\int d\mathbf{r}_k dl_{kz}\exp(-i(\mu\mu_0 l_{kz}\mathbf{k}[\mathbf{r}_k\mathbf{n}_z])/r_k^3)P_1(\mathbf{r}_k,l_{kz}), \tag{18}$$

where $\mathbf{n_x}$ is the unit vector along the X-axis, etc.

Due to the absence of currents along the Z-axis due to the symmetries in the perpendicular plane, the expected microfield can be directed only perpendicular to the layer plane (i.e. along the Z-axis). Thus, the distribution function becomes one-dimensional

$$W(B_z)=\frac{\mu\mu_0}{2\pi}\int dK_z\exp(iK_zH_z)\Pi_{i=1}^{N_x}(1-\frac{16}{15}\sqrt{\frac{\pi^{3/2}}{2}}\frac{K_z<l_{ix}^{3/2}>}{V})^{N_x}$$

$$(1-\frac{16}{15}\sqrt{\frac{\pi^{3/2}}{2}}\frac{K_z<l_{jy}^{3/2}>}{V})^{N_y}=\frac{\mu\mu_0}{2\pi}\int dK_z\cos(K_zH_z)\exp(-(K_zH_{0z})^{3/2}). \tag{19}$$

In the exponential limit, where all mean values of the current element are the same

$$<L_{ix}^{3/2}>=<L_{jy}^{3/2}>=<L^{3/2}>,$$

we get

$$B_{0z}=\mu\mu_0\frac{2^{5/3}\pi^{4/3}}{15^{2/3}}((n_x+n_y)<L^{3/2}>)^{2/3}. \tag{20}$$

Here $n_x+n_y=(N_x+N_y)/V$. Note that if the contacts fill the microchip relatively dense (the percentage of "empty" volume in microchips is less than 50). This way, one finds a rough estimate $H_{0z}\sim<i_e>/l$, where $<i_e>$ is some mean current through one element (and l is the characteristic scale) of microchips. Thus, the magnitude of a stochastic magnetic microfield inside microchips may reach or be above the value of 10^{-6} T, i.e. slightly less than the eart'sh magnetic field. At the same time, the "long tail" of one-dimensional Holtsmark distributions (19) may provide large fluctuations (much) stronger than the earth magnetic induction.

4 Conclusion

The approach to calculate magnetic microfields which was developed in this work, permits to treat several interesting physical situations like plasma displays, microelectronic chips and membranes. The anisotropic geometries corresponding to realistic devices are of special interest. Several examples of anisotropic problems are discussed here in detail. In spite of the rather small magnitudes, the magnetic microfields in all of the considered problems (plasma screens, hydrogen fuel cell membranes, microchips, membranes in living cells) may provide measurable physical effects. Possibly one can use the predicted effects for developing a magnetic field diagnostics, similar to the tomographic tools in medicine and to the quite intriguing methods used by animals for the orientation in the earth's magnetic field. In spite of the fact that the development of a field diagnostics is always directed to the detection of structures, or possibly defects in structures, it needs as a basic tool the knowledge of the stochastic distribution, in order to differ between systematic local structures/defects and stochastic fluctuations.

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