

Pressure Ionization and Transitions in Dense Hydrogen

W. Ebeling¹, H. Hache¹, H. Juranek², R. Redmer^{*2}, and G. Röpke²

¹ Humboldt-Universität zu Berlin, Institut für Physik, Newtonstr. 15, D–12489 Berlin, Germany

² Universität Rostock, Institut für Physik, D–18051 Rostock, Germany

Received 20 March 2005, accepted 25 March 2005

Published online 13 June 2005

Key words Equation of state, dense hydrogen, ionization, dissociation.

PACS 51.30+i, 52.25.Jm, 52.25.Kn, 52.35.Tc

Shock experiments with fluid hydrogen have shown that a transition from insulating behavior to metal-like conductivity occurs at pressures beyond 100 GPa. This requires the development of new methods to describe the transition region of dense plasmas. The traditional approach due to Saha is based on the assumption of chemical equilibrium between charged and neutral components. This is equivalent to minimizing the free energy with respect to the composition. Here we improve an expression for the free energy developed recently to determine Hugoniot curves and isentropes in dense hydrogen and deuterium plasma in the regions of partial dissociation and partial ionization. We show that at high pressures the influence of the excluded volume occupied by neutral species is crucial for the transition to full ionization. We present curves for several thermodynamic functions for the region $5000 \text{ K} < T < 20\,000 \text{ K}$ and $0.6 \text{ g/cm}^3 < \rho < 1 \text{ g/cm}^3$. The influence of the effective radii of the neutral species is crucial in the transition region.

© 2005 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

1 Introduction

The equation of state (EOS) of dense hydrogen in the transition region from low to high ionization is a topic which has initiated many experimental investigations [1–3]. Beginning with the pioneering works of Wigner and Huntington [4], Abrikosov [5], and Ashcroft [6], also numerous theoretical studies were performed; see, e.g., [7–15]. The general conclusion derived from this experimental and theoretical work is that dense hydrogen and deuterium shows a transition to full ionization at temperatures of about $T \simeq (10^3 - 10^4) \text{ K}$ and pressures beyond 1 Mbar (1 Mbar = 100 GPa = 10^{11} Pa). However, many questions concerning the mechanism and type of this transition remain open, for instance, whether it is a first-order phase transition with a discrete jump in density and a corresponding latent heat or a continuous transition [10, 11, 14–17].

Recent shock-wave experiments were able to reach temperatures in the region $T \simeq (10^3 - 10^4) \text{ K}$ and pressures beyond 1 Mbar and provided detailed information on the EOS and the transition to full ionization by measuring simultaneously the conductivity and/or reflectivity [18–24]. For instance, metal-like conductivities have been observed in shock-compression experiments by using a two-stage light gas gun in the fluid domain around 140 GPa and 3000 K [18]. Furthermore, significant discrepancies between the Hugoniot curves derived from laser-driven [19, 20] and magnetically-launched flyer-driven [25] shock-wave experiments have to be stated in the region above 40 GPa. The latter results coincide with theoretical equations of state such as the Sesame tables [8] and molecular dynamics simulations within the generalized gradient approximation of density functional theory [26, 27] as well as with path-integral Monte Carlo simulations [28]. Chemical models using concepts of linear mixing [29, 30], however, are able to reproduce the increased compressibility found in the laser-driven experiments around 100 GPa. Inspection of the reflectivity data along the Hugoniot gives clear evidence for a transition from a nonconducting, molecular state at pressures below 20 GPa to a fully ionized plasma with a reflectivity of about 60–70% [31].

* Corresponding author: e-mail: ronald.redmer@uni-rostock.de, Phone: +49 381 498 6910, Fax: +49 381 498 6912

© 2005 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

Both effects, the transition to metallic-like behavior and the increased compressibility, would substantially change our present understanding of the behavior of hydrogen at ultra-high pressures which is the basis for models of planetary and stellar interiors and inertial confinement fusion studies.

In this paper we give a critical discussion of the transition to high ionization by calculating several thermodynamic functions by means of optimization principles. For instance, the chemical approach first developed by Saha is based on the assumption of chemical equilibrium between charged and neutral species. The set of Saha equations is equivalent to minimize the free energy of the system with respect to the degree of ionization and dissociation, respectively. Recently, we have derived a new expression for the free energy [32, 33] within the framework of the chemical picture. By minimizing the free energy with respect to the degree of ionization α and the degree of dissociation β , we have calculated the isothermal equation of state (EOS), the isentropes, and the Hugoniot curves, see also [34, 35]. In this paper, we study the behavior of the pressure, the free energy, the enthalpy, and the entropy in the transition region.

2 Hydrogen equation of state in the chemical picture

First we present the chemical approach to the free energy of dense hydrogen which was applied recently to temperatures between 2000 and 10 000 K [32, 33]. The effects of pressure dissociation, $\text{H}_2 \rightleftharpoons 2\text{H}$, and ionization, $\text{H} \rightleftharpoons \text{e} + \text{p}$, are taken into account so that the transition from a molecular fluid at low temperatures and pressures through a partially dissociated, warm fluid at medium temperatures of some 1000 K to a fully ionized, hot plasma above 10 000 K can be explained.

In this region, hydrogen consists of two main components, the plasma (electrons and protons) and the neutral fluid (atoms and molecules). Correspondingly the free energy expression for a two-component system of neutral (F_{ne}) and charged particles (F_{pl}),

$$F(V, T, N) = F_{ne}(V, T, N) + F_{pl}(V, T, N), \quad (1)$$

combines results for the fully ionized plasma domain [11] with improved data for the dense, neutral fluid considering pressure dissociation [36, 37]. Both contributions to the free energy are split into ideal and interaction parts. We take into account the interactions in the neutral and in the charged subsystem, respectively, while the interaction between charges and neutrals is accounted for by the reduced volume concept.

Instead of going into details we discuss here only the principal structure of the free energy in the chemical approach as function of the variational parameters. We consider hydrogen at fixed temperature T and total proton density $n = n_p + n_a + 2n_m$, where n_p is the free proton, n_a the atomic, and n_m the molecular number density. For simplicity, the formation of other species as H_2^+ and H^- will be neglected. The condition of charge neutrality requires then that the electron and proton density are always equal, i.e. $n_e = n_p$.

The degrees of ionization α and dissociation β can be defined via the following relations [32]:

$$\alpha = \frac{n_p}{n}, \quad \beta_a = \frac{n_a}{n}, \quad \beta_m = \frac{2n_m}{n}, \quad \beta_d = \frac{n_a}{n_a + 2n_m}. \quad (2)$$

We note that β_a is the relative amount of protons bound in atoms and β_m is that of protons bound in molecules. β_d is the degree of dissociation of molecules into atoms in the neutral subsystem.

The free energy has to be minimized with respect to these variational parameters. A first systematic study of this variational problem was performed by Graboske, Harwood, and Rogers [7]. We take into account free electrons and protons, atoms, and molecules since molecular ions play no role at the high densities we are interested in. Thus, we have only two independent variational parameters, α and one β . Due to the balance equation for the total proton density n , we find the relations

$$\beta_a = \alpha(1 - \beta_d), \quad \beta_m = (1 - \alpha)(1 - \beta_d). \quad (3)$$

It can be shown that atoms appear only in a rather narrow region of the density-temperature plane. Therefore, we may assume in a simplified approximation $\beta_a = 0$ and $\beta_m \simeq 1 - \alpha$, so that α remains as the only free parameter. Since the plasma and the molecular gas are the most relevant reference states for hydrogen, we will use the parameters α (degree of ionization) and β_m (degree of molecule formation) as independent variational parameters.

We start from the free energy per volume

$$f(T, n; x, y) = \frac{F}{V} \quad (4)$$

which has the dimension of an energy density or pressure. Alternatively, also the free energy per proton in units of $k_B T$ can be used as a dimensionless quantity,

$$\phi(T, n; x, y) = \frac{F}{k_B T N_p}. \quad (5)$$

Both thermodynamic quantities depend only on four independent parameters. Instead of the total proton density n we may also use the mass density $\varrho = m_p n$. The variational parameters are the degrees of ionization x and dissociation y . In thermodynamic equilibrium we have $x_0 = \alpha$ and $y_0 = \beta_m$ which have to be distinguished from the running variational parameters.

The equilibrium composition is found by the minimization procedures

$$f(T, n; x, y) \rightarrow \min \quad \text{or} \quad \phi(T, n; x, y) \rightarrow \min. \quad (6)$$

In the special case that the minimum is located inside the simplex, the variation may be replaced by the differential conditions

$$\frac{\delta f}{\delta x} = 0, \quad \frac{\delta f}{\delta y} = 0 \quad \text{or} \quad \frac{\delta \phi}{\delta x} = 0, \quad \frac{\delta \phi}{\delta y} = 0. \quad (7)$$

The minimum values are the real degrees of ionization and dissociation,

$$\alpha = x_{min}, \quad \beta = y_{min}, \quad (8)$$

where β is one of the three dissociation parameters introduced above.

We discuss now in brief the different contributions to the free energy per proton. According to Eq. (1) we assume the decomposition [32],

$$\phi = \phi_{ne} + \phi_{pl}, \quad (9)$$

where the contribution of the neutrals has the form

$$\phi_{ne} = \phi_a^{id} + \phi_m^{id} + \phi_{ne}^{int}. \quad (10)$$

We have performed classical Monte Carlo simulations with several thousand particles in a box for partially dissociated, fluid hydrogen for a grid of temperature and density points in the region $T = (2000 - 10\,000)$ K and $\varrho = (0.2 - 1.1)$ g/cm³. Effective pair potentials of the exponential-6 form were used to model the interactions between the molecules and atoms in the dense fluid. The dissociation equilibrium $\text{H}_2 \rightleftharpoons 2 \text{H}$ was solved by taking into account the correlation parts of the chemical potentials within fluid variational theory [36, 37]. The Monte Carlo data for the interaction contribution can be interpolated accurately within an eight-parameter fit with respect to density and temperature leading to an analytical expression for the free energy density; for details, see [32]. It gives the thermodynamic functions on the basis of density- and temperature-dependent hard sphere radii. For temperatures above 20 000 K, a Carnahan-Starling expression with a fixed hard sphere diameter was used. In the intermediate temperature region we have interpolated between both expressions.

The contribution of the plasma has the structure

$$\phi_{pl} = \phi_e^{id} + \phi_i^{id} + \phi_{ee} + \phi_{ii} + \phi_{ie}. \quad (11)$$

The first terms describe the ideal contributions of the electrons and protons. The interaction contributions of the charged component account for correlations and exchange in the electronic and ionic subsystem, and the electron-ion screening. They are treated in terms of Padé approximations which are based on analytical results for the quantum virial expansion [38].

3 Pressure ionization and reduced volume concept

In this section we study in particular the influence of the spatial effects expressed by different effective radii of the neutral species.

(i) *Repulsive atom-atom interactions.* As pointed out above we apply for temperatures below 10 000 K fluid variational theory considering pressure dissociation [37]. For temperatures above 20 000 K we use the Carnahan-Starling approximation f_{hs} for atoms with an effective hard sphere radius R_{aa} and a packing parameter $\eta_a = \frac{4\pi}{3}R_{aa}^3(n_a + 2n_m)$:

$$f_{hs} = k_B T (n_a + 2n_m) \frac{(4\eta - 3\eta^2)}{(1 - \eta)^2}. \quad (12)$$

We consider molecules formally as consisting of two atoms. This is not a serious approximation since the molecular fraction is small in the temperature region studied in the present work. For the parameter R_{aa} we use here a value of $R \simeq a_B$. A slight modification of this value does not change the results very much.

(ii) *Repulsive interactions between charges and neutrals.* For the ideal free energy density of electrons f_e^{id} we apply Fermi-Dirac statistics. Furthermore, we take into account corrections due to an excluded volume so that the free energy density is given as [40]

$$f_e = n_e k_B T z \left(\frac{N_e \Lambda_e^3}{2V_e} \right). \quad (13)$$

The function $z(x)$ was originally introduced by Zimmermann [39] and interpolates between the low-density limit, $z(x \ll 1) \approx \ln x - 1$, and the high-density limit, $z(x \gg 1) \sim x^{\frac{2}{3}}$; see also [40]. $\Lambda_e = h/\sqrt{2\pi m_e k_B T}$ is the thermal wave length of electrons. The thermal wave lengths of protons, atoms, and molecules (Λ_p , Λ_a , Λ_m) is defined correspondingly. The volume $V_e = \xi_e V$ is the free volume approachable for electrons. The excluded volume factor $\xi_e < 1$ expresses the fact that a part of the total volume V is occupied already by atoms and molecules and, because of Pauli blocking effects, is not accessible for free electrons. At high densities the accessible volume $V_e = \xi_e V$ for the electrons is considerably smaller than the total volume and, therefore, the effective densities are higher and the Pauli pressure increases. A strict theory for the excluded volume effect is not available yet. We will use the following *ansatz* for the excluded volume factor,

$$\xi(\eta_e) = \frac{V_e}{V} = 1 - \eta_e, \quad (14)$$

where $\eta_e = \frac{4}{3}\pi R_{ae}^3(n_a + 2n_m)$ defines the packing fraction of atoms with respect to free electrons, i.e. the radius R_{ae} is the smallest distance to which free electrons can approach atoms.

We have to chose values for this radius. The estimate $R_{ae} = 1.0 a_B$ is based on the naive idea that free electrons cannot be localized within the first Bohr orbit of the hydrogen atom. Several studies within the so-called confined atom model [7] have shown that a hydrogen atom is already destroyed if the wall is placed at $2 a_B$. Therefore, we consider $R_{ae} = 1.5 a_B$ as a reasonable approximation in what follows.

The same arguments apply also for the ions so that the ideal free energy density of protons f_p^{id} can be written as [40]

$$f_p^{id} = n_p k_B T [\ln(n_p \Lambda_p^3) - 1] - n_p k_B T \ln(1 - \eta_i). \quad (15)$$

The first term is the classical expression and the second one describes the excluded volume effect.

4 Composition and thermodynamic functions

The thermodynamic properties are obtained by minimizing the free energy with respect to the degree of ionization and dissociation. The behavior of the degree of ionization α and of molecule formation β_m at $T = 20\,000$ K, $10\,000$ K, and 5000 K is shown in Fig. 1. Most interestingly, the degree of ionization increases abruptly at

a certain density which is known as pressure ionization. The excluded volume factor strongly determines the location of this transition. The corresponding curves for the pressure are displayed in Fig. 2 and show a van der Waals wiggle for $T < 18\,000$ K, pointing to the existence of a pressure-induced phase transition in the transition region. The pressure amounts about $P \simeq 10^{12}$ Pa = 10 Mbar in this transition region, in qualitative agreement with earlier estimates [32,33] and with the transition pressure to a conducting state observed experimentally [18].

In order to study the thermodynamics in the transition region in more detail, several thermodynamic functions were calculated in addition. We present in Fig. 3 the free energy, the internal energy, the enthalpy, and the entropy for a supercritical temperature of $T = 20\,000$ K and a subcritical temperature of $T = 5\,000$ K. We note that the calculations do not include any Maxwell construction for the pressure. At the same position where the pressure shows a van der Waals wiggle, the free and internal energy have a concave (from below) region pointing to the instability leading to the transition.

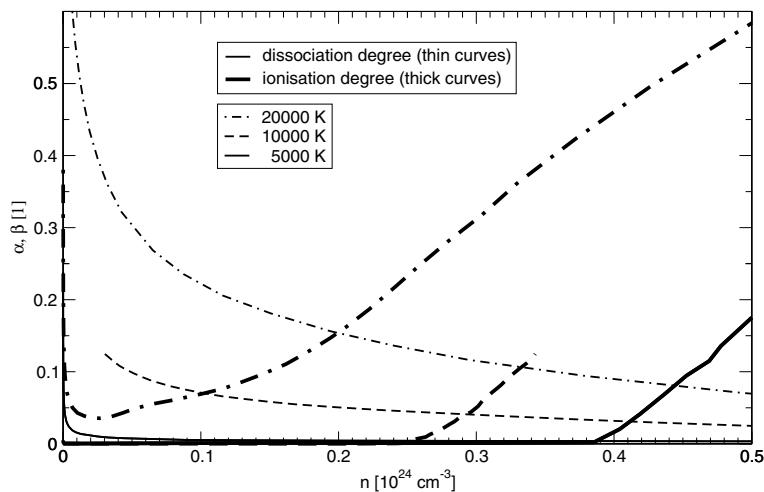


Fig. 1 Degree of ionization α (thick lines) and dissociation β_m (thin lines) in hydrogen for the temperatures 5000 K, 10000 K, and 20000 K as function of the particle density n . The hard core radii are $R_{aa} = R_{ae} = 1.5 a_B$.

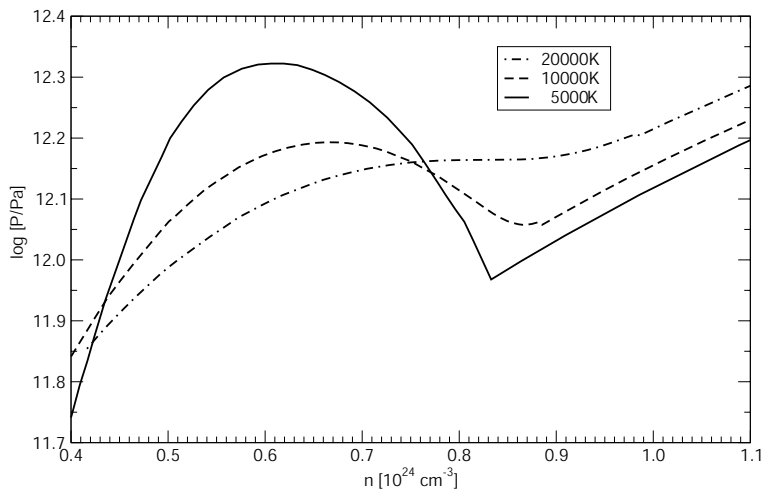


Fig. 2 Pressure in hydrogen in Pa (log-scale) for the temperatures 5000 K, 10000 K, and 20000 K as function of the particle density n . Pronounced van der Waals wiggles are to be seen for $T < 20\,000$ K; the critical temperature is estimated to be $T_{cr} \simeq 18\,000$ K.

The slope of the curves for the thermodynamic functions shown in Figs. 3 is similar to those obtained recently by Juraneck within fluid variational theory [41]. The 20000 K-isotherm for TS increases monotonically, while

the 5000 K-isotherm shows a wiggle which is connected with the behavior of α and β_m . The free energy per proton F has a minimum for both temperatures, and the internal energy per proton U as well. The enthalpy per proton $G = U - TS$ has a wiggle at the lower temperature 5000 K that stems from the behavior of TS .

The temperature dependence of this transition to full ionization, also known as plasma phase transition, is demonstrated in Fig. 4. We see that the transition density decreases with temperature and that the density jump is rather small, less than 10%. For $T = 5000$ K we find a discontinuity between $(4.17 - 4.57) \times 10^{23} \text{ cm}^{-3}$, and for $T = 10\,000$ K between $(4.27 - 4.68) \times 10^{23} \text{ cm}^{-3}$. Furthermore, the related entropy jump is also rather small. We can give an estimate as $T\Delta S/\Delta V \simeq 10^{11} \text{ Pa} = 1 \text{ Mbar}$. From the Clausius-Clapeyron equation

$$\frac{dP}{dT} = \frac{\Delta S}{\Delta V} \quad (16)$$

follows that the coexistence line should have a small negative slope. This agrees with earlier findings of Beule *et al.* [42] and Saumon and Chabrier [14].

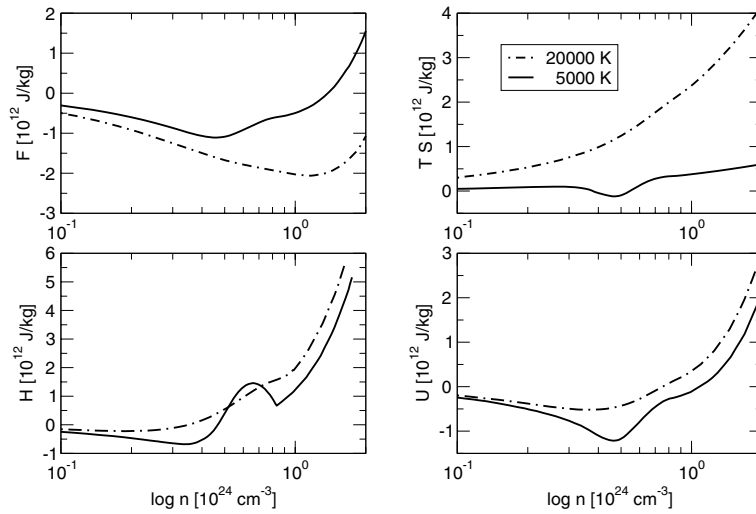


Fig. 3 Free energy F , entropy TS , enthalpy H , and internal energy U in hydrogen for a supercritical temperature of 20000 K and a subcritical temperature of 5000 K as function of the particle density n .

5 Discussion

The present work is based on the chemical description of plasmas in combination with a minimization procedure for the free energy with respect to the degree of ionization α and the degree of molecule formation β_m . The plasma is considered as a mixture different species: free electrons, free ions (protons), atoms, and molecules. The free energy of this mixture is calculated based on

- the quantum-statistical theory of fully ionized plasmas leading to first-order nonideality corrections at low or high densities, respectively,
- Padé approximations connecting the low-density region with the high-density region,
- fluid variational theory for neutral hydrogen which proved to be a very good approximation for the temperature region $T < 10\,000$ K,
- Carnahan-Starling approximations for the neutral component in the region of higher temperatures $T > 20\,000$ K, which model the neutral particles as hard spheres.

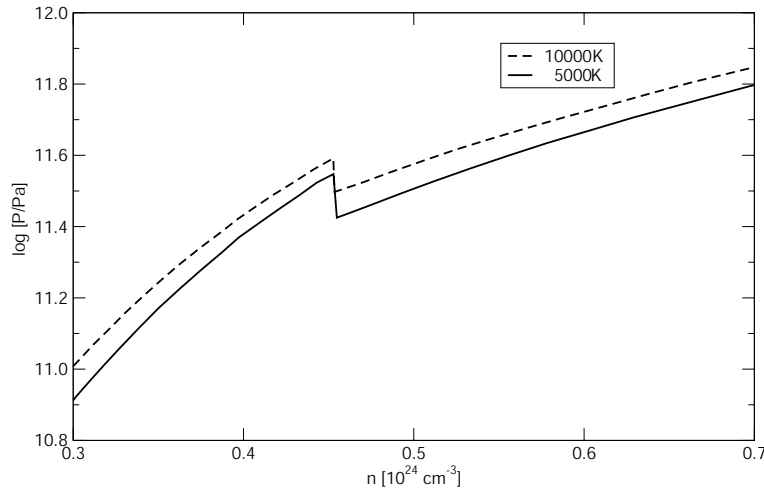


Fig. 4 Pressure isotherms for 5000 K and 10 000 K for a wide range of densities in Pa (log-scale). Unlike the calculations for Fig. 2 we have chosen now $R_{ae} = 2a_B$ which gives a lower transition pressure. The isotherms show a sharp van der Waals wiggle, pointing to a first-order phase transition at the density $n = 4.5 \times 10^{23} \text{ cm}^{-3}$.

By combining these different theoretical expressions we have derived the free energy of the mixture, where the composition parameters (the percentage of protons which are free or bound in atoms or molecules) appear as free variational parameters. The real composition is obtained by minimization of the free energy. This procedure is carried out by means of a MATHEMATICA program, which is (in principle) able to yield compositions for any given density and temperature as well as all thermodynamic function of interest. The progress is based on the formulae given in Refs. [32, 42], including also reduced volume effects.

This work is devoted to hydrogen. In order to use the present hydrogen EOS also for deuterium, mass scaling can be applied for the interpolation formula of the interaction contributions. It is assumed that the same particle numbers for hydrogen and deuterium lead to the same degree of dissociation and to identical interaction contributions to the thermodynamic function of the neutral fluid for a given temperature.

Acknowledgements We acknowledge helpful discussions with V.E. Fortov, W.-D. Kraeft, B. Militzer, and V.B. Mintsev.

References

- [1] V.F. Grigoriev, S.B. Kormer, O.L. Mikhailova, A.P. Tolochko, V.D. Urlin, Pis'ma v Zh. Eksp. Teor. Fiz. **16**, 286 (1972); Zh. Eksp. Teor. Fiz. **69**, 743 (1975).
- [2] H.K. Mao, R.J. Hemley, Rev. Mod. Phys. **66**, 671 (1994).
- [3] V.E. Fortov, I.T. Yakubov, *Physics of Nonideal Plasmas* (Hemisphere Publ., New York, 1990).
- [4] E.P. Wigner, H.B. Huntington, J. Chem. Phys. **3**, 764 (1935).
- [5] A.A. Abrikosov, Sov. Phys.-JETP **39**, 1798 (1960); **41**, 565 (1961); **45**, 2038 (1962).
- [6] N.W. Ashcroft, Phys. Rev. Lett. **21**, 1748 (1968).
- [7] H.C. Graboske, Jr., D.J. Harwood, F.J. Rogers, Phys. Rev. **186**, 210 (1969).
- [8] G.I. Kerley, Los Alamos Scientific Laboratory Report LA-4776 (Los Alamos, 1972).
- [9] E.G. Brovman, Yu. Kagan, A. Xholas, Sov. Phys.-JETP **61**, 2429 (1971).
- [10] W. Ebeling, Physica **73**, 573 (1984).
- [11] W. Ebeling, W. Richert, phys. stat. sol. (b) **128**, 467 (1985); Phys. Lett. A **108**, 80 (1985); Contrib. Plasma Phys. **25**, 1 (1985).
- [12] P. Haronska, D. Kremp, M. Schlanges, Wiss. Z. Univ. Rostock MNR **36**, 98 (1987).
- [13] W. Ebeling, A. Förster, R. Redmer, T. Rother, M. Schlanges, in: Invited papers of the XVIIIth ICPIG in Swansea 1987, p. 40.
- [14] D. Saumon, G. Chabrier, Phys. Rev. Lett. **62**, 2397 (1989); Phys. Rev. A **44**, 5122 (1991); *ibid.* **46**, 2084 (1992).
- [15] M. Schlanges, M. Bonitz, A. Tschtschjan, Contrib. Plasma Phys. **35**, 109 (1995).
- [16] A.A. Likalter, Phys. Rev. B **53**, 4386 (1996); J. Exp. Theor. Phys. **86**, 598 (1998) [Zh. Eksp. Teor. Fiz. **113**, 1094 (1998)].
- [17] W. Ebeling, H. Norman, J. Stat. Phys. **110**, 861 (2003).

- [18] S.T. Weir, A.C. Mitchell, W.J. Nellis, Phys. Rev. Lett. **76**, 1860 (1996); W.J. Nellis, S.T. Weir, A.C. Mitchell, Phys. Rev. B **59**, 3434 (1999).
- [19] L.B. Da Silva, P. Celliers, G.W. Collins, K.S. Budil, N.C. Holmes, T.W. Barbee, Jr., B.A. Hammel, J.D. Kilkenny, R.J. Wallace, M. Ross, R. Cauble, A. Ng, G. Chiu, Phys. Rev. Lett. **78**, 483 (1997).
- [20] G.W. Collins, L.B. Da Silva, P. Celliers, D.M. Gold, M.E. Foord, R.J. Wallace, A. Ng, S.V. Weber, K.S. Budil, R. Cauble, Science **281**, 1178 (1998).
- [21] V.Ya. Ternovoi, A.S. Filimonov, V.E. Fortov, S.V. Kvitov, D.N. Nikolaev, A.A. Pyalling, Physica B **265**, 6 (1999).
- [22] S.I. Belov, G.V. Boriskov, A.I. Bykov, R.I. Il'kaev, N.B. Luk'yanov, A.Y. Matveev, O.L. Mikhailova, V.D. Selemir, G.V. Simakov, R.F. Trunin, I.P. Trusov, V.D. Urlin, V.E. Fortov, A.N. Shuikin, JETP Letters **76**, 433 (2002).
- [23] V.E. Fortov, V.Ya. Ternovoi, M.V. Zhernokletov, M.A. Mochalov, A.L. Mikhailov, A.S. Filimonov, A.A. Pyalling, V.B. Mintsev, V.K. Gryaznov, I.L. Iosilevskii, J. Exp. Theor. Phys. **97**, 259 (2003).
- [24] S.K. Grishetshkin, S.K. Gruzdev, V.K. Gryaznov, M.V. Zhernokletov, R.I. Il'kaev, I.L. Iosilevskii, G.N. Kashinzeva, S.I. Kirshanov, S.F. Manatshkin, V.B. Mintsev, A.L. Mikhailov, A.B. Mezhevov, M.A. Mochalov, V.E. Fortov, V.V. Khrustalev, A.N. Shuikin, A.A. Yukhimtshuk, Pis'ma v Zh. Exp. Teor. Fiz. **80**, 452 (2004).
- [25] M.D. Knudson, D.L. Hanson, J.E. Bailey, C.A. Hall, J.R. Asay, W.W. Anderson, Phys. Rev. Lett. **87**, 225501 (2001).
- [26] T.J. Lenosky, S.R. Bickham, J.D. Kress, L.A. Collins, Phys. Rev. B **61**, 1 (2000).
- [27] G. Galli, R.Q. Hood, A.U. Hazi, F. Gygi, Phys. Rev. B **61**, 909 (2000).
- [28] B. Militzer, D. Ceperley, Phys. Rev. Lett. **85**, 1890 (2000).
- [29] N.C. Holmes, M. Ross, W.J. Nellis, Phys. Rev. B **52**, 15 835 (1995).
- [30] M. Ross, Phys. Rev. B **58**, 669 (1998).
- [31] P.M. Celliers, G.W. Collins, L.B. Da Silva, D.M. Gold, R. Cauble, R.J. Wallace, M.E. Foord, B.A. Hammel, Phys. Rev. Lett. **84**, 5564 (2000).
- [32] D. Beule, W. Ebeling, A. Förster, H. Juranek, S. Nagel, R. Redmer, G. Röpke, Phys. Rev. B **59**, 14 177 (1999).
- [33] D. Beule, W. Ebeling, A. Förster, H. Juranek, R. Redmer, G. Röpke, Contrib. Plasma Phys. **39**, 21 (1999).
- [34] R. Redmer, H. Juranek, S. Kuhlbrodt, V. Schwarz, Z. Phys. Chem. **217**, 783 (2003).
- [35] V. Bezkrovniy, M. Schlanges, D. Kremp, W.-D. Kraeft, Phys. Rev. E **69**, 061204 (2004); V. Bezkrovniy, V.S. Filinov, D. Kremp, M. Bonitz, M. Schlanges, W.-D. Kraeft, P.R. Levashov, V.E. Fortov, *ibid* **70**, 057401 (2004).
- [36] A. Bunker, S. Nagel, R. Redmer, G. Röpke, Phys. Rev. B **56**, 3094 (1997); Contrib. Plasma Phys. **37**, 115 (1997).
- [37] H. Juranek, R. Redmer, J. Chem. Phys. **112**, 3780 (2000); H. Juranek, R. Redmer, Y. Rosenfeld, *ibid.* **117**, 1768 (2002).
- [38] W. Stolzmann, W. Ebeling, Phys. Lett. A **248**, 242 (1998).
- [39] R. Zimmermann, *Many-Particle Theory of Highly Excited Semiconductors*, (B.G. Teubner Verlagsgesellschaft, Leipzig, 1988), pp. 150.
- [40] W. Ebeling, A. Förster, V.E. Fortov, V.K. Gryaznov, A.Ya. Polishchuk, *Thermophysical Properties of Hot Dense Plasmas* (B.G. Teubner Verlagsgesellschaft, Stuttgart and Leipzig, 1991).
- [41] H. Juranek, PhD Thesis (University of Rostock, 2004).
- [42] D. Beule, W. Ebeling, A. Förster, H. Juranek, R. Redmer, G. Röpke, Phys. Rev. E **63**, 060202(R) (2001).